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STRUCTURAL DAMAGE CHARACTERIZATION USING FREE DECAYS

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A new technique for the identification and characterization of damage in structures is presented. The technique employs a newly developed non-exponential transient decay model which relies on a statistical description of the structure's modal characteristics. The resulting technique provides an estimate of both the severity and extent of the damage without reliance on detailed modal information. The technique is supported by numerical simulations of damaged truss structures.

1. INTRODUCTION

Many techniques have been proposed for the non-invasive identification and characterization of damage in structures. Most concentrate on correlating damage with changes in the frequency domain characteristics of the structure, primarily natural frequencies. (The review by Salawu [1] provides a comprehensive discussion of these techniques). These approaches are fundamentally inhibited, however, by the trade-off between spatial resolution and the accuracy of frequency domain predictions. High spatial resolution requires the use of high order modal information which is difficult to accurately model while low order modal characteristics are accurately predictable but provide poor spatial resolution.

This paper presents a time domain damage identification technique which relies on observed free structural responses and their corresponding energy decay characteristics. The technique offers high spatial resolution, independent of the structure's modal details, providing estimates of both the extent and severity of damage in the structure. High spatial resolution is possible because the structure's modal characteristics are modelled statistically, allowing the use of high order modal responses.

The technique is inspired by prior research to identify and characterize damage in polycrystals using transient energy decays. These techniques assume proportional, visco-elastic damping and relate microstructural damage, which increases internal friction [2], to energy decay rates [3–5]. The proportional damping assumption inherently assumes that the damage in the polycrystal is uniformly distributed throughout the material volume. This technique is impractical in structural applications because damage is often spatially localized. Additionally, a variety of mechanism unrelated to damage exists which might affect the energy decay rate of a typical in-service structure. More recently, the author has presented an additional decay technique, more suitable to structural applications, which identifies and characterizes non-uniform damage distributions in visco-elastic solids. This technique relies on a non-exponential, transient decay model for non-proportionally damped systems [6]. While adding considerable complexity, non-exponential decays also present an opportunity because, when properly modelled, they can be used to inversely determine damage characteristics as demonstrated in ultrasonic systems [6].

This paper extends the ultrasonic technique to the identification and characterization of structural damage which increases dissipation, such as corroded or loosely fastened connections, by exploiting the characteristics of non-exponential decays. The technique is developed for application in structures which have a fraction of their connections damaged such that they provide a local viscous energy sink. Specifically, section 2 presents a theoretical model describing viscous, non-exponential, structural decay. The experimental use and range of application of the proposed technique is then discussed briefly in section 3. Numerical experiments which support the application of the proposed technique are presented in section 4 while section 5 provides some concluding remarks.

2. THEORY

The proposed damage characterization technique is based on a model for the transient decay of the total energy of vibration in a structure. The structures are modelled by a system of coupled, linear, second order ordinary differential equations written in terms of the structure's displacement vector $\{x\}$, mass matrix $[m]$, stiffness matrix $[k]$, and damping matrix $[c]$,

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{0\}. \tag{1}$$

This model is valid for the free response of all viscously damped, linear structural systems including those with complicated, non-proportional damping. It should be emphasized that it is the damping matrix $[c]$, which is used to characterize the damage in the structure. Specifically, viscous losses are added to each damaged degree of freedom resulting in a damping matrix which describes the intensity and distribution of damage throughout the structure.

Predicting transient decays in non-proportionally damped systems is complicated because the system's undamped modal basis does not uncouple the damped equations of motion (1), and, in fact, results in both complex valued eigenvalues, $\lambda_l = \omega_l + \mathrm{i}\beta_l$, and eigenvectors $\{u_l\}$ [7]. The dissipation characteristics of such a system can, however, be explored by calculating the power flow in the structure. A real-valued expression for the power flow in the structure can be derived by premultiplying the equation of motion (1), by the complex conjugate of the system's velocity vector $\{\dot{x}^*\}^{\mathrm{T}}$,

$$\{\dot{x}^*\}^{\mathrm{T}}[m]\{\ddot{x}\} + \{\dot{x}^*\}^{\mathrm{T}}[c]\{\dot{x}\} + \{\dot{x}^*\}^{\mathrm{T}}[k]\{x\} = \{0\} \tag{2}$$

and adding the resulting equation (2), to its complex conjugate. Realizing that

$$\{\dot{x}^*\}^{\mathrm{T}}[m]\{\ddot{x}\} + \{\dot{x}\}^{\mathrm{T}}[m]\{\ddot{x}^*\} = \frac{\mathrm{d}}{\mathrm{d}t}[\{\dot{x}^*\}^{\mathrm{T}}[m]\{x\}],$$

$$\{\dot{x}^*\}^{\mathrm{T}}[k]\{x\} + \{\dot{x}\}^{\mathrm{T}}[k]\{x^*\} = \frac{\mathrm{d}}{\mathrm{d}t}[\{x^*\}^{\mathrm{T}}[k]\{x\}], \tag{3}$$

a real-valued expression for the power flow in the system can be written as

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}[\{\dot{x}^*\}^{\mathrm{T}}[m]\{\dot{x}\} + \{x^*\}^{\mathrm{T}}[k]\{x\}] + \{\dot{x}^*\}^{\mathrm{T}}[c]\{\dot{x}\} = \{0\} \tag{4}$$

Equation (4) broadly characterizes the power flow in the system by equating the time rate of change of the kinetic and potential energy with the rate of energy dissipation through viscous mechanisms.

If the motion of the system is restricted to the nth mode, $\{x\} = \{u_n\}\exp(\mathrm{i}\lambda_n t)$, an expression for the nth decay rate β_n can be derived from equation (4) assuming $[m]$, $[c]$ and $[k]$ are symmetric:

$$\beta_n = \frac{1}{2}\frac{\{u_n^*\}^{\mathrm{T}}[c]\{u_n\}}{\{u_n^*\}^{\mathrm{T}}[m]\{u_n\}}, \tag{5}$$

which is valid for all viscous damping matrices $[c]$; no other limitation is imposed.

While equation (5) provides an expression for all decay rates in the structural system, and therefore is capable of characterizing non-exponential decay, it is of limited use because employing it in this fashion would require complete knowledge of the structures' modal characteristics ($\{u_n\}$, ω_n).

A more efficient description of the decay rates in the structural system can be obtained statistically. The general approach, most recently presented by Burkhardt and Weaver [3], is to treat modal vectors, $\{u_n\}$, as uncorrelated, random Gaussian vectors. It is further assumed that the viscous dissipation intensity is a constant value, c, for all damaged degrees of freedom but, importantly, not all degrees of freedom are damaged. As a result, each damaged location in the structures is "equally damaged" but arbitrarily located. Additionally, assuming that light damping yields real-valued modal vectors, an expression for the expected statistical distribution of modal decay rates, $p(\beta)$, can be derived [3]:

$$p(\beta) = \frac{\beta^{\frac{B}{2}-1}e^{-\beta\left(\frac{mN}{c}\right)}}{(c/mN)^{B/2}\Gamma(B/2)}, \tag{6}$$

in terms of c, the viscous damping intensity, m, the system's mass density, and B and N, the number of damped degrees of freedom and the total number of degrees of freedom in the system respectively.

With an expression for the probability density function of decay rates, equation (6), the ensemble average energy decay of the structure, $d(t)$, composed of a collection of modes with decay rates drawn from the derived distribution, can be calculated. Assuming the total initial energy in the structure, E_0, is equally distributed among the modes,

$$d(t) = E_0\int_0^\infty p(\beta)\,\mathrm{e}^{-2\beta t}\,\mathrm{d}\beta = E_0\left[1 + \left(\frac{2c}{mN}\right)t\right]^{-B/2}, \tag{7}$$

a power-law description for the non-exponential decay process is recovered, characterized by two independent parameters

$$\frac{c}{mN} \quad \text{and} \quad \frac{B}{2}. \tag{8}$$

The first parameter, c/mN, characterizes the intensity of the structural damage through the viscous damping factor c, while the second parameter, $B/2$, provides an estimate of the extent of the structural damage through B, the number of damaged degrees of freedom.

Now, with an expression for the non-exponential decay process, equation (7), it is possible to exploit its form to determine the two free parameters, equation (8), from an observed energy decay. The parameters can be determined in one of the two ways, either through a non-linear curve fit of equation (7) or by a linear regression analysis of

$$-\left[\frac{\mathrm{d}}{\mathrm{d}t}\{\ln[\mathrm{d}(t)]\}\right]^{-1} = \frac{2}{B} + \left(\frac{2c}{mN}\right)t, \tag{9}$$

which follows directly from equation (7). Determination of the two free parameters provides an immediate estimate of the intensity of the damage through the viscous damping parameter, c, as well as the spatial extent of the damage through the parameter B.

3. APPLICATION

Practical application of the developed technique for the characterization of structural damage has limitations on its application beyond the explicit assumptions made during its derivation. General limitations on energy decay methods have already been discussed in detail by Weaver [9]. In summary, the developed technique is applicable provided the structure's decay is reverberant and involves the participation of numerous modes. Reverberance, which requires that many system transits occur within a characteristic decay time, provides validity to the assumption that modal amplitudes are Gaussian random variables. Numerous modes, and therefore numerous decay rates, are desirable because the derived decay model is an ensemble average result averaged over all possible decay rates. Consequently, numerous modes and decay rates participating in the observed decay are more likely to match the derived decay model.

An additional limitation on the technique is imposed by signal duration. The system under interrogation must posses a decay duration which is sufficient to manifest curvature. On short time scales all energy decays will appear exponential. It is necessary to observe the signal over a time scale in which the difference in the decay rates of the individual modes, which may be small, becomes significant. Practically speaking, the curvature will become evident when the slope of the logarithm of the energy decay is significantly different from its initial slope.

Consider the slope of the logarithm of decay, equation (7), normalized by its slope at time zero:

$$\mu(t) = \frac{(\mathrm{d}/\mathrm{d}\tau)\{\ln[\mathrm{d}(t)]\}}{(\mathrm{d}/\mathrm{d}\tau)\{\ln[\mathrm{d}(0)]\}} = \left[1 + \left(\frac{2c}{mN}\right)t\right]^{-1}. \tag{10}$$

Now define a characteristic curvature time, $t_{1/2}$, as the time required for the slope of the logarithm of decay to fall to half its original value:

$$\mu(t_{1/2}) = 1/2 \Rightarrow t_{1/2} = \frac{mN}{2c}. \tag{11}$$

The meaning of this expression is more clear in terms of a non-dimensional time $\tau = 2\langle\beta\rangle t$, where $2\langle\beta\rangle = cB/mN$ is the mean energy decay rate. Then,

$$\mu(\tau) = \left[1 + \left(\frac{2}{B}\right)\tau\right]^{-1} \Rightarrow \tau_{1/2} = \frac{B}{2}, \tag{12}$$

which indicates that a signal duration of $B/2$ energy decay times is required for the slope of the logarithm of decay to fall to half its original value. Consequently, as the number of damaged degrees of freedom rises longer signal durations are required to observe the non-exponential decay.

4. NUMERICAL EXPERIMENTS

As a test of the proposed damage characterization technique numerical experiments were performed on nominally undamped, square, planar trusses of unit dimension with 90 degrees of freedom, $N = 90$, Figure 1. The truss is composed of a series of nominally identical elastic rods constrained to move in the plane of the structure. To model the complexity of a typical structural system the positions of the truss joints are slightly perturbed, thus varying the length of the individual rods. Three separate damage scenarios are considered with viscous dampers added to either 4, 8 or 12 degrees of freedom, $B = 4, 8, 12$. Additionally, the corner nodes of the structure are fixed to prevent motion in the plane.

The common characteristics of the simulated systems are as follows: bar mass density = 1, bar cross-sectional area = 1, modulus of elasticity = 1, and intensity of viscous damage, $c = 1$. These system characteristics are chosen because they result in a system transit time of 1 s. Consequently, a wave travelling in the system will take 1 s to cross the

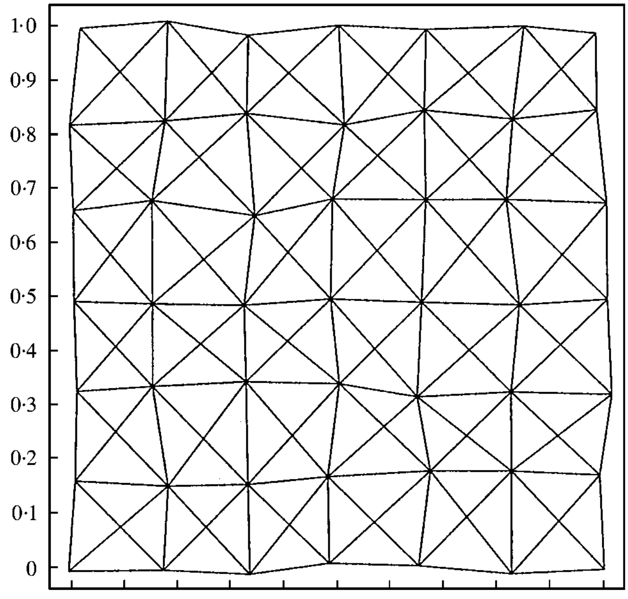

Figure 1. Sample truss configuration.

unit width of the system. In addition to these common characteristics each simulated truss has B degrees of freedom chosen randomly to be "damaged" by the inclusion of a viscous damper. Once the configuration of the truss was determined its mass $[m]$, damping $[c]$, and stiffness $[k]$, matrices were constructed using standard finite element techniques. Global mass matrices were assembled using a consistent rather than lumped formulation.

The transient response of the truss to an initial displacement of the central node was determined by integrating the equation of motion (1), using the MATLAB function "ode23t" through $t = 100$. This first required rewriting the governing equation using a state-space formulation because of the non-proportional characteristics of the damping matrix [7]. The resulting governing equation in state space is a "traditional" equation of motion amenable to standard integration techniques such as those offered by MATLAB.

The resulting structural response was interrogated at two interior nodes, location 1 and location 2, with the energy density of the nodes recorded as a function of time. A third energy density signal was generated by spatially averaging the two independent receiver locations, avg sample. Additionally, the total energy present in the truss was recorded as a function of time, total energy. The four recorded broadband decay signals were then fit to the natural logarithm of non proportional decay model, equation (7), using the MATLAB function "nlinfit." Additionally, no configuration averaging was performed; all results presented were for single truss realizations.

The results for the three damage scenarios, $c = 1{\cdot}0$, $B = 4, 8, 13$, are shown in Figures 2–4 with the best-fit, non-proportional decay model shown as a solid black line. Additionally, estimates of the two significant curve-fit parameters are shown with an overbar, $\bar{B}$ and $\bar{c}$.

In each damage scenario, $c = 1{\cdot}0$, $B = 4, 8, 12$, the proposed technique yields close estimates for both the number of damped degrees of freedom B, and damage intensity, c. This is particularly significant considering that virtually no averaging was performed. The single spatial average that was performed did not yield particularly more accurate results.

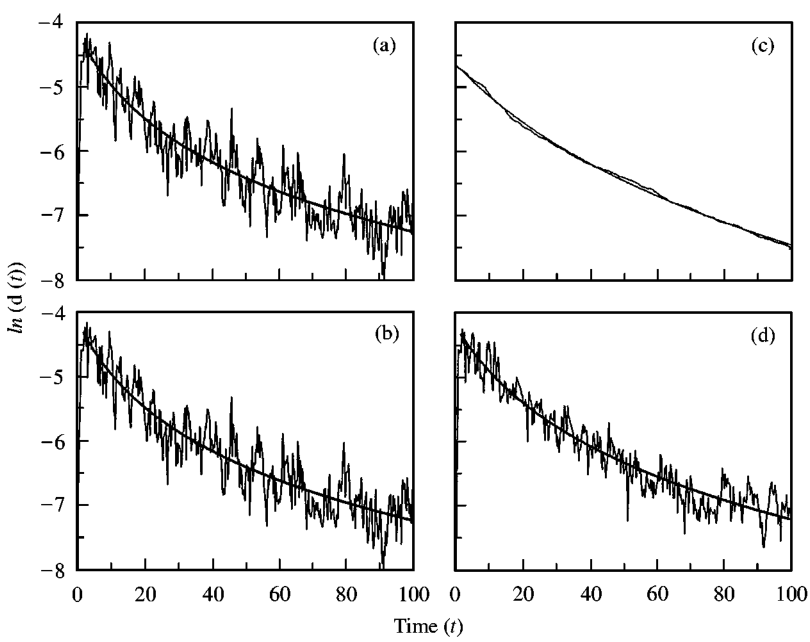

Figure 2. Non-proportional structural decays and model fits: $B = 4$, $c = 1$. (a) Location 1: $\bar{B} = 2{\cdot}8$, $\bar{c} = 2{\cdot}4$. (b) Location 2: $\bar{B} = 3{\cdot}6$, $\bar{c} = 1{\cdot}3$. (c) Total energy: $\bar{B} = 4{\cdot}5$, $\bar{c} = 0{\cdot}7$. (d) Avg. sample: $\bar{B} = 3{\cdot}6$, $\bar{c} = 1{\cdot}7$.

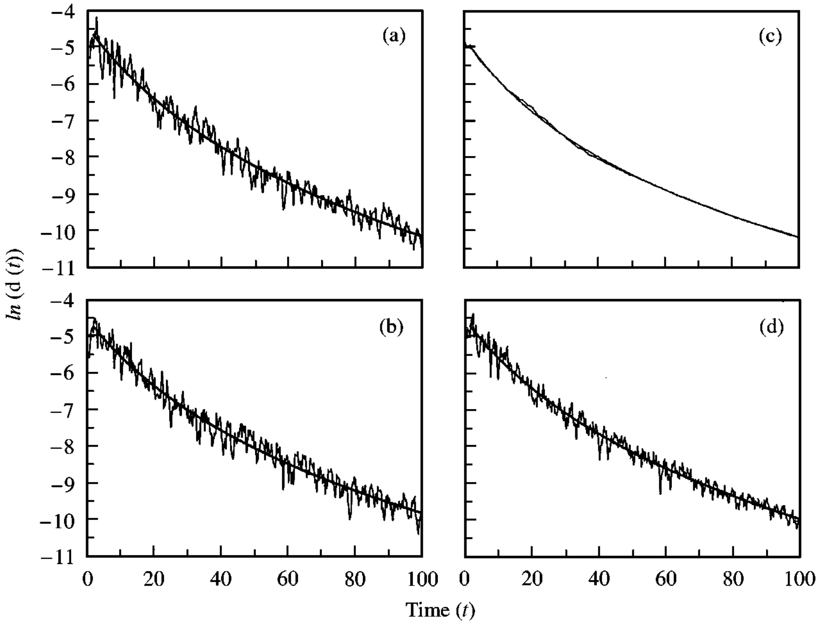

Figure 3. Non-proportional structural decays and model fits: $B = 8$, $c = 1$. (a) Location 1: $\bar{B} = 8{\cdot}0$, $\bar{c} = 0{\cdot}9$. (b) Location 2: $\bar{B} = 7{\cdot}2$, $\bar{c} = 1{\cdot}0$. (c) Total energy: $\bar{B} = 6{\cdot}6$, $\bar{c} = 1{\cdot}2$. (d) Avg. sample: $\bar{B} = 7{\cdot}6$, $\bar{c} = 0{\cdot}9$.

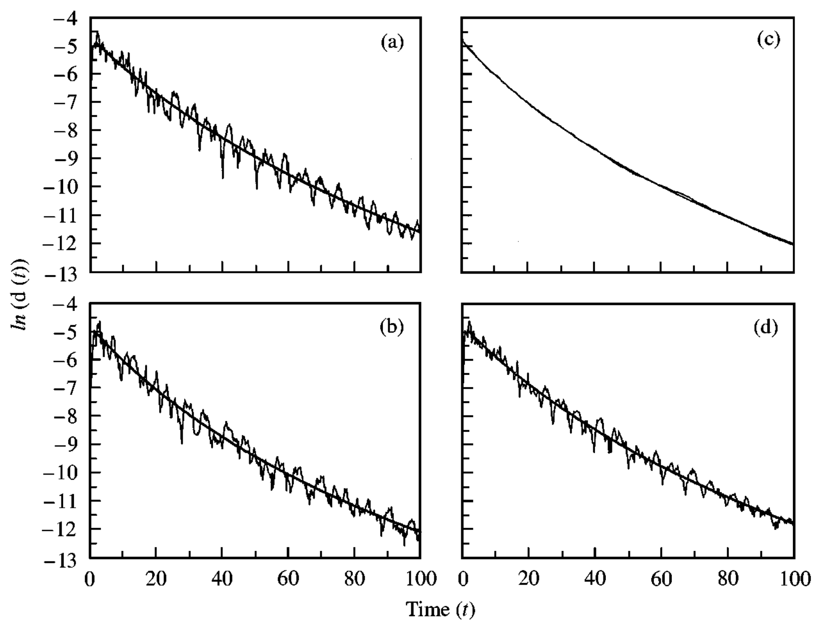

Figure 4. Non-proportional structural decays and model fits: $B = 12$, $c = 1$. (a) Location 1: $\bar{B} = 14{\cdot}1$, $\bar{c} = 0{\cdot}5$. (b) Location 2: $\bar{B} = 12{\cdot}8$, $\bar{c} = 0{\cdot}6$. (c) Total energy: $\bar{B} = 13{\cdot}2$, $\bar{c} = 0{\cdot}6$. (d) Avg. sample: $\bar{B} = 13{\cdot}6$, $\bar{c} = 0{\cdot}5$.

This is not unexpected, for averaging to be significant it must occur over multiple decay rates. Spatial averaging provides no new information concerning decay rates. The response at each location results from the same modes and therefore the same decay rates.

5. CONCLUSIONS

A theoretical model for the non-exponential decay of viscously damaged structures has been derived and shown to be a useful tool for the determination of both the intensity and extent of viscous damage in structures. Importantly, the technique is independent of modal details and, therefore, is insensitive to modelling details and service-related changes to the structure. This is in contrast to many current damage characterization techniques which require detailed model information and modal characteristics.

The technique also has several significant limitations including restriction to viscous damage mechanisms and an inability to spatially locate the damage. Future work will concentrate on exploring the effect of non-viscous damage mechanisms on the technique as well as experimentally testing its effectiveness.

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